

## An Improved Procedure for Combining Day-of-Launch Atmospheric Flight Loads

July 1999

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Prepared for

SPACE AND MISSILE SYSTEMS CENTER  
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2430 E. El Segundo Boulevard  
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Contract No. F04701-93-C-0094

Engineering and Technology Group

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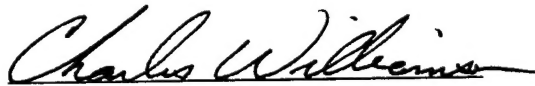
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This report was submitted by The Aerospace Corporation, El Segundo, CA 90245-4691, under Contract No. F04701-93-C-0094 with the Space and Missile Systems Center, P. O. Box 92960, Los Angeles, CA 90009-2960. It was reviewed and approved for The Aerospace Corporation by R. W. Fillers, Principal Director. The project officer is Maj. Charles R. Williamson.

This report has been reviewed by the Public Affairs Office (PAS) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nationals.

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A handwritten signature in cursive script, reading "Charles R. Williamson". The signature is written in dark ink and is positioned above the printed name and title.

Maj. Charles R. Williamson  
Project Officer

# REPORT DOCUMENTATION PAGE

Form Approved

OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE July 1999	3. REPORT TYPE AND DATES COVERED Final
4. TITLE AND SUBTITLE An Improved Procedure for Combining Day-of-Launch Atmospheric Flight Loads			5. FUNDING NUMBERS F04701-93-C-0094
6. AUTHOR(S) J. F. Binkley, J. B. Clark, C. E. Spiekermann			8. PERFORMING ORGANIZATION REPORT NUMBER TR 99(1534)-3*
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) The Aerospace Corporation 2350 E. El Segundo Boulevard El Segundo, CA 90245			
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Space and Missile Systems Center Air Force Materiel Command 2430 E. El Segundo Boulevard Los Angeles Air Force Base, CA 90245			10. SPONSORING/MONITORING AGENCY REPORT NUMBER SMC-TR-00-22
11. SUPPLEMENTARY NOTES			
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited			12b. DISTRIBUTION CODE
13. ABSTRACT (Maximum 200 words) An improved procedure for combining launch vehicle atmospheric flight load contributors is presented. This new procedure produces load-to-allowable value ratios with lower bias and less variance than previously used procedures. It is expected that the new procedure will increase launch availability without reducing launch reliability.			
14. SUBJECT TERMS Loads Combination Equation, capability ratio, central limit theory			15. NUMBER OF PAGES 17
			16. PRICE CODE
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT

## **Abstract**

An improved procedure for combining launch vehicle atmospheric flight load contributors is presented. This new procedure was applied to loads from a heavy lift launch vehicle and was found to produce load-to-allowable value ratios with lower bias and less variance than a widely-used procedure. It is expected that the new procedure will increase launch availability without reducing launch reliability.



## Contents

Abstract.....	1
Nomenclature .....	5
1. Introduction .....	7
2. DOL Implementation of LCE.....	9
3. DOL Implementation of the BILCE.....	11
4. Accuracy of the BILCE .....	13
5. Conclusions .....	15
References .....	17

## Figures

1.	LCE method to compute limit load .....	10
2.	Differences between the Monte Carlo CR and the LCE and BILCE CRS at 48 launch vehicle stations at mach 1.58 for a heavy lift launch vehicle .....	13
3.	Additional CR difference plots at various mach numbers.....	14

## Table

1.	Predicted Bending Moments (Million Inch-Pounds).....	3
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## Nomenclature

BILCE	Bivariate Integration LCE
$C_K$	Coefficient of order K Hermite polynomial
CLT	Central Limit Theorem
CR	Capability ratio
DOL	Day of launch
$f_x$	Density function of a random variable x
$H_K$	Order K Hermite polynomial
$L_{ALLOW}$	Structural allowable load, lb or in-lb
$L_{AXIAL}$	Axial load, lb or in-lb
$L_{EQ}$	Equivalent axial load, lb or in-lb
LCE	Loads combination equation
$M_p$	Pitch bending moment, lb or in-lb
$M_T$	Combined yaw and pitch bending moment, lb or in-lb
$M_Y$	Yaw bending moment, lb or in-lb
$m_3$	Third central moment
$N(\mu, \sigma)$	Normal random variable with mean $\mu$ and standard deviation $\sigma$
P	Random variable representing $M_p$
Pr	Probability
RSS	Root sum of squares
$U(a,b)$	Uniform random number with minimum a and maximum b
X	Random variable representing $M_x$
Y	Random variable representing $M_Y$
$\mu_x$	Mean of random variable X
$\sigma_x$	Standard deviation of random variable X





## 1. Introduction

Most launch vehicles can only achieve the desired level of structural reliability by restricting the winds through which the vehicle is allowed to fly. This restriction is accomplished by first analytically flying the vehicle through wind profiles that are measured just prior to launch, and calculating altitude histories of angles of attack, dynamic pressure, rigid body acceleration, and engine gimbal angles, among others. These time histories are then used to establish static-aeroelastic and other day-of-launch calculated loads, which are then combined with the pre-day-of-launch calculated loads to obtain the total load.<sup>1,2</sup> Since many of the individual loads have random characteristics, a statistical load enclosure of the total load is computed. This total load enclosure is then compared to the vehicle allowable strength, and if it is exceeded, the vehicle is not launched. If sufficient time is available before the end of the launch window, the entire process is repeated for subsequent wind profiles. If there is not enough time, the launch attempt is aborted, and the vehicle is prepared for the next available launch window.

For the launch vehicle core structure, it is typical to express the comparison between the day-of-launch calculated load enclosures and the allowable values as capability ratios (CRs); i.e.,

$$CR = \frac{\text{Equivalent Axial Load}}{\text{Structural Allowable Load}} = \frac{L_{EQ,MAX}}{L_{ALLOW}} \quad (1)$$

The structural allowable load,  $L_{ALLOW}$ , is determined prior to the day of launch, usually with structural testing and appropriate factors of safety. The equivalent axial load,  $L_{EQ}$ , is determined from an evaluation of the stress at a cross section of the launch vehicle. It is a function of the resultant bending moment,  $M_T$ , obtained from the combination of the yaw and pitch bending moment components,  $M_Y$  and  $M_P$ . It also depends on the axial load,  $L_{AXIAL}$ , and the radius to the point on the launch vehicle cylinder wall;

$$L_{EQ,MAX} = L_{AXIAL,MAX} + \frac{2M_T}{\text{Radius}} \quad (2)$$

The combined pitch and yaw bending moments,  $M_T$ , are computed by combining a set of statistically varying loads from a variety of sources. To ensure, with a high probability, that the predicted equivalent axial load is not exceeded during atmospheric flight, a load enclosure of  $M_T$  is used in Eq. 2 to compute the CRs. The load enclosure is usually computed at a 99.7 percent enclosure, 90 percent confidence, level.<sup>3</sup>

The enclosure load is computed by combining loads obtained from analyses performed to simulate the various atmospheric flight load contributors. Static-aeroelastic (STEL) loads analyses are performed to establish the loads that are due to that portion of the vehicle's angle of attack that vary relatively slowly with time. The STEL load is a function of the day-of-launch winds and the vehicle steering profile. Gust analyses are performed to establish the response of the vehicle, and its payload, to the turbulence that might be encountered on any given flight. Buffet analyses are performed to establish the loads due to the dynamic response of the vehicle/payload system to shock waves, flow separation due to changes in vehicle geometry, and the interaction between the two. In addition, other analyses are often performed to estimate loads due to items such as vehicle load alleviation steering, autopilot noise, wind measurement error, changes in day-of-launch winds from the time they are measured to when the vehicle is to be launched, and vehicle dispersions from the nominal parameters used in the analyses.

The most accurate method of computing the enclosure load is by Monte Carlo simulation.<sup>4</sup> Due to time constraints, Monte Carlo simulation is currently not a realistic computational option during day-

of-launch operations. An analysis and launch decision is required within a matter of a few minutes, and a Monte Carlo simulation requires considerably more time. In order to quickly calculate CRs on the day of launch, a computationally efficient analytical procedure is necessary.

A widely used analytical method for computing the enclosure load uses what are referred to as Loads Combination Equations (LCEs).<sup>1,2,5</sup> Enclosure loads and the resulting CRs computed using the LCEs have been shown to be biased on the conservative side. This bias reduces launch availability. Reduction factors have been incorporated into the LCEs to reduce the conservative bias. However, this can also, on occasion, lead to underpredicting the CRs.

This paper presents the derivation of a new procedure, referred to as the Bivariate Integration Load Combination Equation (BILCE), that has less bias and variance than other LCE procedures. Comparisons of the performance of the BILCE and LCE are made by using both to derive load CRs from several heavy lift launch vehicle load datasets, and comparing these to load CRs computed using a Monte Carlo simulation.

## 2. DOL Implementation of LCE

An example of an actual LCE input load dataset from the day of launch of a heavy lift launch vehicle is given in Table 1. Only the STEL, lack-of-wind persistence, and wind measurement error loads are established on the day of launch, since these depend on the day-of-launch winds and steering profile. The remaining loads are computed prior to the day of launch. Most of the loads are assumed to follow a normal distribution, except the buffet and autopilot loads, which are assumed to follow a Rayleigh distribution,<sup>6</sup> and the gust load, which is assumed to follow a gamma distribution.<sup>7</sup>

Table 1. Predicted Bending Moments (Million Inch-Pounds)

Source	Computation	Distribution	Yaw Plane		Pitch Plane	
			Mean	Standard Deviation	Mean	Standard Deviation
STEL	DOL	-	97.53	0	635.61	0
Wind Persistence	DOL	Normal	86.11	141.87	68.95	109.37
Wind Measurement Error	DOL	Normal	0	82.59	0	91.08
Trajectory Dispersion	Pre-DOL	Normal	0	100.18	0	96.28
Aerodynamic	Pre-DOL	Normal	0	3.16	0	19.91
Maneuvering	Pre-DOL	Normal	167.49	24.56	147.55	19.86
Wind Data Gap	Pre-DOL	Normal	0	0	0	0
			Root Sum of Squares		Root Sum of Squares	
Buffet	Pre-DOL	Rayleigh	119.72		141.22	
Auto Pilot	Pre-DOL	Rayleigh	0		0	
			Response to 30-ft/sec Gust		Response to 30-ft/sec Gust	
Gust	Pre-DOL	Gamma	1976.58		1909.37	

A pictorial of how the enclosure load is determined is given in Figure 1. In the figure,  $\Sigma M_y$  and  $\Sigma M_p$  are the yaw and pitch vector sums (summed means + RSS of dispersed terms), while R is the 99.7 percent enclosure load. The steps to compute the enclosure load are: 1) Compute the mean and dispersed term of each load in the yaw and pitch directions. The dispersed term is the load's 99.7 percentile minus its mean. 2) Reduce the means and dispersed terms of the gust, buffet, autopilot, trajectory, and maneuvering loads by ten percent, as a typical value. This is the reduction factor obtained from a Monte Carlo simulation (Ref. 1). 3) Add the sum of the means (excluding STEL) with the RSS of the dispersed terms in the yaw and pitch planes (Fig. 1a). 4) Translate the yaw and pitch vector sums to the STEL's yaw and pitch vector load (Fig. 1b). 5) Construct an ellipse such that major and minor axis vertices are the translated yaw and pitch vector sums (Fig. 1c). 6) The enclosure load is the magnitude of the point on the ellipse that is the maximum distance from the origin (Fig. 1d).

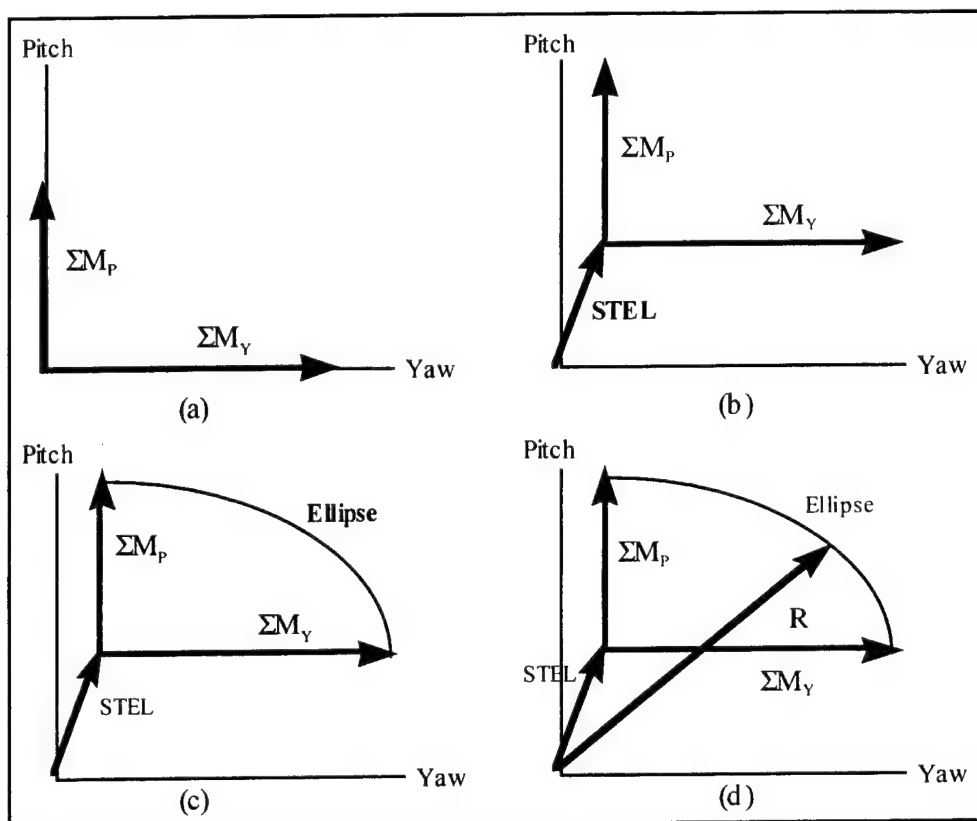


Figure 1. Illustrates the LCE method to compute the limit load.  $\Sigma M_y$  and  $\Sigma M_p$  are the yaw and pitch vector sums (i.e., summed mean + RS of dispersed terms). R is the 99.7 percent enclosure load.

Further details of the methodology used in this LCE can be found in Ref. 1. The LCE has sufficed for many years and many launches by various heavy and medium lift launch vehicles. Although it has served its purpose, the approach has a certain inherent bias and variability that are due to the following reasons: 1) The mean sum added to the RSS of the dispersed terms (steps 3 and 4) is not a 99.7 percent enclosure for the sum of non-normal loads. An example to illustrate this is the sum of two independent  $U(0,1)$  random variables. The 99.7 percentile of the sum is 1.92, whereas the mean sum added to the RSS of the dispersed terms is 1.70. 2) The magnitude of the point on the ellipse that is the maximum distance from the origin (steps 5 and 6) is not the 99.7 percent bound of the load magnitude, even if the vertices of the ellipse are the true 99.7 percent load bounds. An example here is the RSS of two independent  $N(0,1)$  random variables where the center of the ellipse is the origin. The 99.7 percentile of the RSS is 3.41, whereas the magnitude of the line from the origin to the point on the ellipse furthest from the origin is 3.89. Even with these approximations, Monte Carlo simulations have shown that the LCE generally performs as intended. However, it is now possible to combine the various load contributors in a more rigorous manner. It is the purpose of this paper to introduce such a procedure.

### 3. DOL Implementation of the BILCE

The enclosure load of the bending moment is the value  $R$  such that

$$\Pr\left(\sqrt{Y^2 + P^2} \leq R\right) = 0.997 \quad (3)$$

where  $Y$  and  $P$  are random variables representing the sum of the yaw and pitch bending moments,  $M_Y$  and  $M_P$ . Given that  $Y$  and  $P$  have density functions,  $f_Y$  and  $f_P$ , Eq. (3) can be rewritten as

$$\iint_{\sqrt{u^2 + v^2} \leq R} f_Y(u) f_P(v) du dv = 0.997 \quad (4)$$

Due to the non-normal loads (buffet, autopilot, and gust) and the numeric difficulties in convoluting their density functions with the other bending moment densities, computing  $f_Y$  and  $f_P$  and consequently the exact value of  $R$  in a timely manner during day-of-launch operations is time consuming.

Since  $Y$  is the sum of several statistically independent loads, from the Central Limit Theorem, its density function,  $f_Y$ , is approximately normal with mean,  $\mu_Y$ , the sum of the individual load's means, and standard deviation,  $\sigma_Y$ , the RSS of the individual load's standard deviations. This approximation is sufficient as long as the dominating loads are approximately normal. Unfortunately, two significant load contributors, buffet and gust, have Rayleigh and gamma distributions, respectively, which are both highly skewed to the right. The Central Limit Theorem approximation can be improved<sup>8</sup> by modifying the normal density approximation so that its higher central moments are equal to the higher central moments of  $Y$ . The modified density approximation can be written as

$$f_Y(u) \approx \frac{1}{\sigma_Y \sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{u - \mu_Y}{\sigma_Y}\right)^2\right) \cdot \left[1 + \sum_{k=1}^{\infty} C_k H_k\left(\frac{u - \mu_Y}{\sigma_Y}\right)\right] \quad (5)$$

where  $H_k(x)$  is a Hermite polynomial of order  $k$ , and  $C_k$  are coefficients determined by equating the central moments of  $Y$  with the density approximation. Equating only the first three moments, we find

$$C_1 = C_2 = 0 \text{ and } C_3 = \frac{m_3}{6\sigma_Y^3} \quad (6)$$

where  $m_3$  is the third central moment of  $Y$ ; i.e.,  $[E(Y - \mu_Y)^3]$ . Since

$$H_3(x) = x^3 - 3x \quad (7)$$

Eq. (5) can be written as

$$f_Y(u) \approx \frac{1}{\sigma_Y \sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{u - \mu_Y}{\sigma_Y}\right)^2\right) \cdot \left[1 + \frac{m_3}{6\sigma_Y^3}\left(\frac{(u - \mu_Y)^3}{\sigma_Y^3} - \frac{3(u - \mu_Y)}{\sigma_Y}\right)\right] \quad (8)$$

The approximate density function of P is determined in an analogous manner. Using Eqs. (4) and (8), the steps to compute the BILCE enclosure load are: 1) Compute the mean, standard deviation, and 3<sup>rd</sup> moment (skewness) of each load in the yaw and pitch direction. These statistics may be computed analytically or by Monte Carlo techniques for the pre-day-of-launch loads. 2) Sum the means, variances, and 3<sup>rd</sup> moments to determine  $\mu$ ,  $\sigma$ , and  $m_3$ . 3) Approximate the density functions,  $f_Y$  and  $f_P$ , of the bending moment sums, Y and P, using Eq. (8). 4) The enclosure load is the value R in Eq. (4) found by using Romberg numerical integration (Ref. 2) and the approximate density functions,  $f_Y$  and  $f_P$ .

Though higher moments can be used to approximate  $f_Y$  and  $f_P$ , for the launch vehicle flights examined, the first three moments were adequate to account for the skewness as a result of the buffet and gust loads.

#### 4. Accuracy of the BILCE

The accuracies of the LCE and the BILCE are assessed by comparing their CRs with CRs computed using a Monte Carlo simulation. Details of the Monte Carlo simulation are given in Refs. 3, 4, and 9. Comparisons are made at several launch vehicle stations and flight Mach numbers from nine heavy lift launch vehicle flights.

A comparison of the CRs for one of the flights at Mach 1.58 is shown in Fig. 2. The marks that overlay the top horizontal line are CR differences (Monte Carlo CR - LCE CR) derived at 48 launch vehicle stations. The bottom marks are CR differences at the same stations using the BILCE.

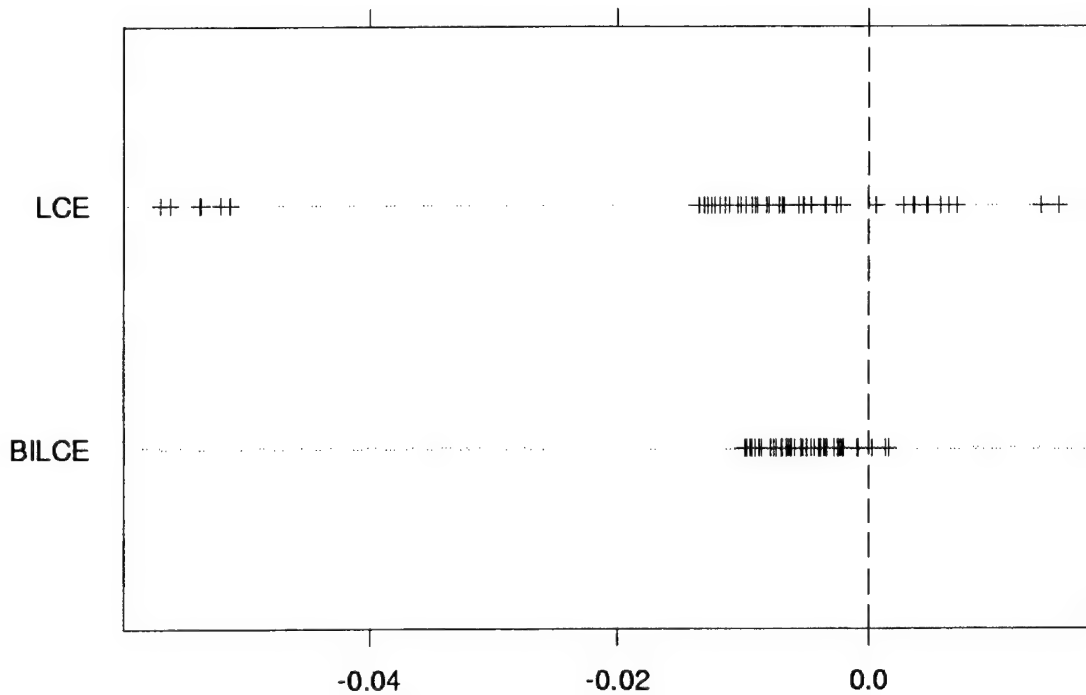


Figure 2. Marks indicate the difference between the Monte Carlo CR and the LCE and BILCE CRS at 48 launch vehicle stations at mach 1.58 for a heavy lift launch vehicle. The improved accuracy of the BILCE compared to the LCE can be seen.

The reduction in variation, using the BILCE compared to the LCE, demonstrates the improved accuracy of the method. For the BILCE, the lack of points far to the left of zero indicates fewer overpredictions of the enclosure load compared to the LCE, providing the potential for making fewer unnecessary "No-Go" launch decisions by using the BILCE. The lack of points far to the right indicates the potential to also make fewer "Go" launch decisions with lower than the desired statistical enclosure.

The CR comparisons for the same flight at the other Mach numbers studied are given in Fig. 3. Results are similar to the Mach number 1.58 values; CR accuracy is improved using the BILCE. CR comparisons for the remaining eight heavy lift launch vehicle flights show similar results.



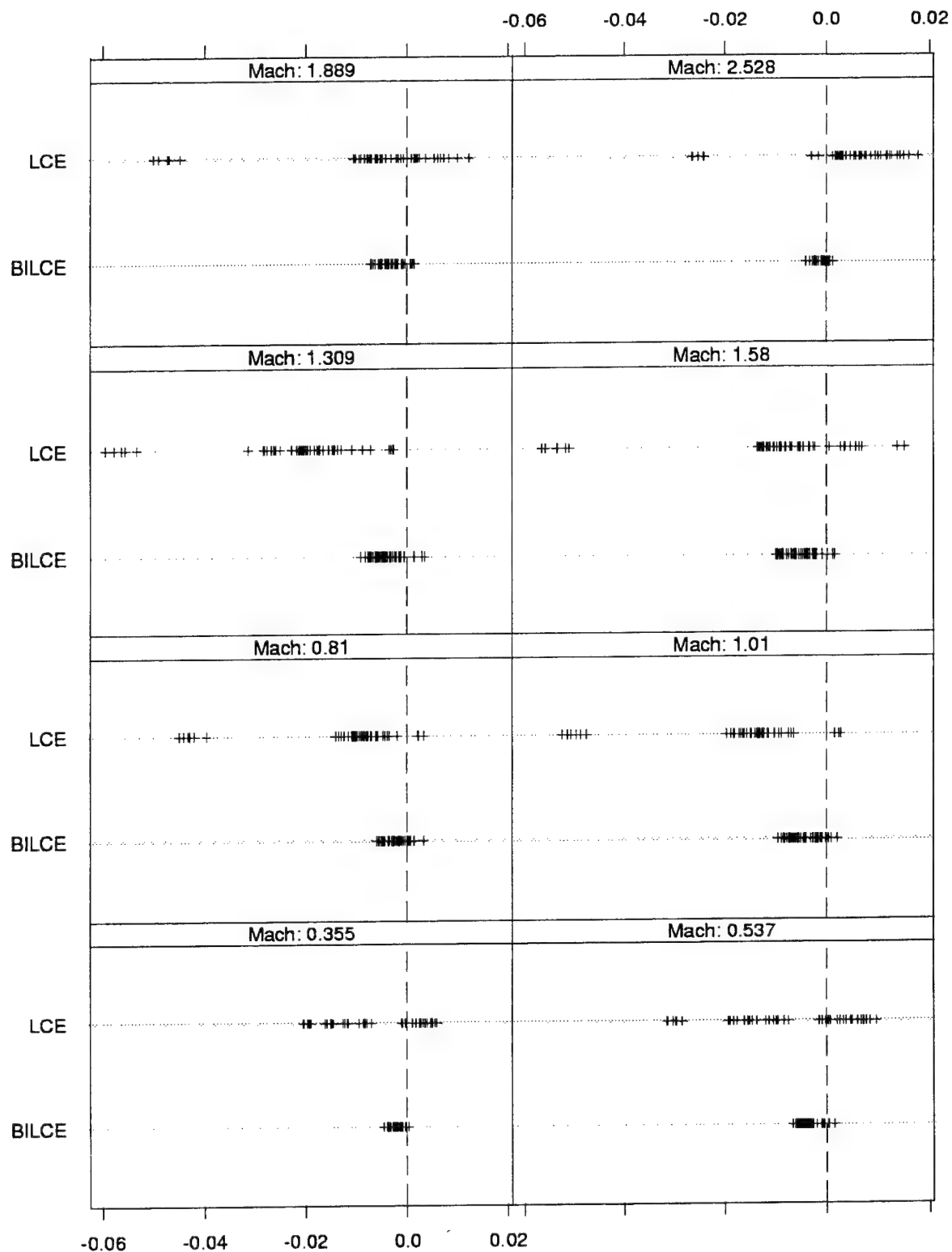


Figure 3. Additional CR difference plots at various mach numbers (Fig. 2 included on right in second row from top). Marks indicate the difference between the Monte Carlo CR and the LCE and BILCE CRS at 48 launch vehicle stations for a heavy lift launch vehicle. Each plot shows the improved accuracy the BILCE compared to the LCE.

## **5. Conclusions**

A refined procedure for combining atmospheric flight loads to compute load-to-allowable ratios has been presented. It is shown that this procedure produces CRs with less bias and variation than a previously-used procedure. As a result, day-of-launch implementation of the proposed procedure should increase launch availability without reducing launch reliability.



## References

1. Macheske, V. M., Womack, J. M., and Binkley, J. F., "A Statistical Technique for Combining Launch Vehicle Atmospheric Flight Loads," AIAA Paper 93-0755, Jan. 1993.
2. Houbolt, J. C., "Combining Ascent Loads, NASA Space Vehicle Design Criteria (Structures)," NASA SP-8099, 1972.
3. Kabe, A. M., "Design and Verification of Launch and Space Vehicle Structures," AIAA Paper 98-1718, April 1998.
4. Womack, J. M., and Binkley, J. F., "A Statistical Technique for Combining Launch Vehicle Loads During Atmospheric Flight," Aerospace Report No. TOR-0091(6530-06)-2, Aug. 1991.
5. NASA, "Combining Ascent Loads," NASA Document SP-8099, May 1972.
6. Spiekermann, C. E., Kabe, A. M., "Statistical Combination of Launch Vehicle Gust and Buffet Atmospheric Flight Loads," AIAA Paper 98-2010, April 1998.
7. NASA, "A Bivariate Gamma Probability Distribution with Application to Gust Modeling," NASA Technical Memorandum No. TM-82483, July 1982.
8. Papoulis, A., Probability, Random Variables, and Stochastic Processes, 2nd Edition, McGraw-Hill Book Company, 1984, pp. 194-200.
9. Book, R. A., "Booster Load Analysis Tool (BLAST) Version 1 User's Guide," Aerospace Report No. TOR (6530-06)-3, July 1991.

